ratio of $A_{ex}/A^* = 2.8$ and a throat diameter of 1.63 in. Longitudinal slots were cut as shown to open up about half the perimeter at the throat. These slots extended a constant width both upstream and downstream of the venturi throat. The actual circumferential suction orifice was formed by blocking off all but a short length of the longitudinal slots with tape. The venturi was housed in a tube which formed a plenum for the suction orifice. The pumping connection was through a hose which fitted over a pipe as shown. Outside the wind-tunnel, this hose was connected to another section of pipe containing a valve and flow-metering orifice. The orifice was constructed according to Ref. 1 and has an orifice diameter of 1.48 in., an orifice diameter d to pipe diameter D ratio of 0.60 and 1 D and 1/2 D pressure connections. The orifice flow coefficient K for air and helium flows was in the range 0.66–0.67 and 0.67–0.69, respectively. Pressures were measured at the venturi throat, in the plenum and across the orifice. Provision was made to connect the intake pipe through a pressure-reducing orifice to a pair of helium tanks, manifolded together. The use of helium as the pumped gas provided a density ratio of about 7 with respect to the wind-tunnel stream. The experiment was performed in the MIT Wright Brothers Wind Tunnel at 100 mph.

Experimental Data

The pumping characteristics of the venturi are shown in Fig. 2 in nondimensional form. The pumped flow is given in terms of a volume flow coefficient.

$$C_Q = Q/u_\infty A^*$$

where

Q = the volumetric flow rate
u_∞ = the speed of the airstream
A^* = the cross-sectional area of the venturi throat

The pressures are given in terms of standard pressure coefficients

$$C_p = (p - p_\infty)/q_\infty$$

where

p = the measured pressure in the plenum
p_∞ = the static pressure in the wind tunnel
q_∞ = the dynamic pressure ($\rho u_\infty^2/2$) in the wind tunnel

The reduced data is shown for four longitudinal locations of the suction orifice with the circumferental slot $l/D^* =$ 0.62 diameters long. For two configurations ($X/D^* = 1.25$ and 1.88), the air and helium data are shown together and are seen to be in agreement for a given configuration. The pressure coefficient at the venturi throat without injected flow was measured to be $C_p = -3.3$.

A cross plot of all the data taken with a suction slot 0.62 diameters long is shown in Fig. 3 and illustrates the variation of the pressure coefficient with slot location for three values of the volume flow coefficient, C_Q. The optimum location is seen to vary from one diameter to 1.5 diameters downstream of the throat as C_Q increases from 0.1 to 0.20.

Configurations with narrower slots and with asymmetric injection were tested and showed poorer performance than the basic slot configuration.

Conclusions

These tests show that the pumping characteristics of a venturi are invariant with respect to the density of the pumped fluid if the pumping rate is expressed in volumetric terms, the optimum location of a circumferential orifice is of the order of one diameter downstream of the throat and varies with the design flow rate, and a suction-pressure corresponding to $C_p = -1$ (pressure drop seen by the system essentially doubled) is achieved if the pumped volume is limited to 15% of the reference flow through the venturi throat.

Reference

[1] *ASME Power Test Code, Supplement on Instruments and Apparatus,* New York, 1959, Pt. 5, Chap. 4.

The Dynamics of Blade Pitch Control

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Nomenclature

C = chord of blade, ft
C_M = steady aerodynamic moment coefficient
n = subscript identifying nth blade
0 = subscript indicating initial state
r = spanwise position along blade, ft
S = subsidiary variable of the Laplace transformation, rad/sec
t = time, sec
x,y,z = rotating Cartesian coordinates, blade principal inertia axes
D_x = unsteady aerodynamic damping coefficient in pitch, ft-lb-sec/rad
I_x,I_y,I_z = blade principal mass moments of inertia, slug-ft^2
K = gain constant of actuator, lb
K_x = virtual spring constant of centrifugal force field in pitch, ft-lb-sec^2/rad^2
M_{xA} = aerodynamic pitching moment, ft-lb
M_{xM} = mechanical pitching moment, ft-lb
N = number of blades
R = blade span, radius of rotor, ft
V = axial velocity in propeller-rotor state, ft/sec
Y = actuator reference input, ft
σ = blade geometric pitch angle, rad
λ = aerodynamic inflow ratio, ratio of axial inflow velocity to blade rotational tip speed
ρ = density of air, slug/ft^3
τ = actuator time constant, sec
ψ = blade azimuth angle, rad
$\omega_x,\omega_y,\omega_z$ = angular velocity components in blade rotating, principal coordinate system, rad/sec
Ω = steady angular velocity of rotor, rad/sec
1,2 = subscripts referring to longitudinal and lateral control directions
$\overline{(\ )}$ = average value of ( ), Laplace transform of ( )
$\dot{}$ = differentiation with respect to time
$'$ = differentiation with respect to azimuth

Introduction

ADVANCED rotorcraft such as the modern helicopter and convertible aircraft utilizing tiltable propeller-rotors frequently employ stability augmentation and gust alleviation devices which require that the pitch settings of the rotor blades be changed both collectively and cyclically in

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a transient manner to alter both rotor thrust magnitude and direction. In the case of cantilever or hingeless blade designs, precise regulation and control of the blade tip path plane with respect to the axis of rotation is of paramount importance because of the very large moments of structural origin which are transferred to the aircraft. Consequently inclusion of the dynamics of blade transient pitch changes is an essential part of both the rotor subsystem and complete aircraft flight control system design and analysis. In the development which follows, it is seen that the system, in general, is nonlinear due to the large aerodynamic inflow and pitch angles in the propeller state. However application of a first-order equilibrium calculation and a second-order perturbation to the appropriate Euler equation of motion leads to a transfer function for collective pitch changes, and a transfer matrix for cyclic pitch changes. In both cases the transfer relationships are found to be strongly dependent on aerodynamic inflow and initial pitch settings. In the case of collective pitch changes system behavior is found to vary from that of a heavily damped oscillator in the hovering helicopter state to a cascaded integrator and time constant process in the propeller-rotor cruise condition. In the case of cyclic pitch changes system behavior ranges from that of a pair of coupled, damped oscillators in the hovering helicopter state to that of an unstable system with a rapidly divergent oscillation in the propeller-rotor cruise condition. Internal stabilization in this latter case is essential, if transient cyclic pitch changes are to be employed in the aircraft flight control system. The principal component of such an internal cyclic pitch stabilization system is seen to be proportional control action on swash-plate angular position feedback.

Analysis

Consider a rotating blade where the mechanical pitch change axis is at the quarter chord points of the airfoil sections and which coincides with the x principal inertia axis of the blade. The y and z principal inertia axes are inclined through the angle σ with respect to the plane of steady rotation and the axis of steady rotation, respectively. The angular velocity of the blade has the steady rotation component Ω, and the instantaneous pitching velocity component $\dot{\sigma}$ about the x principal axis. The equation of motion follows from the Euler equations[1] as

$$I_x \dot{\omega}_x + (I_z - I_y)\omega_z \omega_y = M_{xA} + M_{xM} \quad (1)$$

$$\omega_x = \dot{\sigma} \quad (2)$$

$$\omega_y = \Omega \sin[\sigma_0 + \sigma(t)] \quad (3)$$

$$\omega_z = \Omega \cos[\sigma_0 + \sigma(t)] \quad (4)$$

where M_{xm} is the externally applied mechanical moment of force and M_{xa} is the aerodynamic moment, which we treat as a steady moment due to any camber which may be present and an unsteady pitch damping part due to plunging of the aft neutral point arising from rotation about the quarter chord point.[2] Apparent mass effects are neglected due to the very large density of rotor, prop-rotor, and propeller blades compared to air. Finally we employ a strip theory assumption and integrate over the blade span assuming the axisymmetric flow state of propellers, prop-rotors, and hovering helicopter rotors. The steady aerodynamic moment follows as

$$M(xA_0) = \frac{1}{2}\rho \int_0^R C_M C^2 (V^2 + r^2\Omega^2) dr = 1/6 \rho \bar{C}_M \bar{C}^2 R^3 \Omega^2 (1 + 3\lambda^2) \quad (5)$$

$$M(xA_1) = -\frac{\pi}{8}\rho\dot{\sigma}\int_0^R C^3(V^2 + r^2\Omega^2)^{1/2}dr = -\frac{\pi}{16}\rho\bar{C}^3R^2\Omega\left\{\begin{matrix}(1+\lambda^2)^{1/2} \\ +\lambda ln_e[\frac{1+(1+\lambda^2)^{1/2}}{\lambda}]\end{matrix}\right\}\dot{\sigma} \quad (6)$$

If we now separate the mechanical moment M_{xm} and the pitch angle σ into their steady and time dependent parts, and impose the condition that while the initial pitch angle may be large the unsteady part is relatively small, we obtain the following equations of motion

$$M(xM_0) \cong \frac{1}{2}\Omega^2(I_z - I_y)\sin 2\sigma_0 - \frac{1}{6}\rho\bar{C}_M\bar{C}^2R^3\Omega^2(1 + 3\lambda^2) \quad (7)$$

$$M(xM_1)(\Psi) \cong \Omega^2(I_x\sigma'' + D_x\sigma' + K_x\sigma) \quad (8)$$

$$D_x = \frac{\pi}{16}\rho\bar{C}^3R^2\{(1+\lambda^2)^{1/2} + \lambda ln_e[\frac{1+(1+\lambda^2)^{1/2}}{\lambda}]\} \quad (9)$$

$$K_x = (I_z - I_y)\cos 2\sigma_0 \quad (10)$$

The solution proceeds by solving for the static equilibrium values of $M(xm_0)$ and σ_0 for a given set of design parameters C,R,Ω, etc., and inflow operating parameter λ. This then becomes the basis for the solution of the dynamical behavior of $\sigma(t)$ and its control.

The dynamics of collective pitch changes which alter thrust level by changing the pitch of all N blades simultaneously only need involve a single mechanically coupled actuator. As a first approximation the various types of electro-mechanical and hydro-mechanical actuators may be treated as a simple time constant process[3] where the transfer function is

$$\frac{\bar{M}_{\alpha M}(S)}{\bar{Y}(S)} = \frac{K}{\tau S + 1} \quad (11)$$

and the over-all transfer function for collective pitch changes of an N blade system becomes

$$\frac{\bar{\sigma}(s)}{\bar{Y}(s)} = \frac{(K/N)}{(\tau s + 1)(I_x S^2 + D_x\Omega S + K_\alpha\Omega^2)} \quad (12)$$

In the case of cyclic pitch changes, where longitudinal and lateral control forces and moments are desired from a rotor or prop-rotor, the blades might be linked to a swash-plate mechanism with longitudinal and lateral actuation in a nonrotating frame.[4] In this important case the instantaneous pitch angle of the nth blade is given by

$$\sigma_n(\psi) = \sigma_1(\psi)\sin\psi_n + \sigma_2(\psi)\cos\psi_n \quad (13)$$

$$\psi_n = \Omega t + (2\pi/N)(n-1), \ n = 1,2,\ldots,N \quad (14)$$

The mechanical moments of force to be applied by the two actuators in the nonrotating frame are then given by

$$M_1 = \sum_{n=1}^{N} M(xM_n)\sin\psi_n \quad (15)$$

$$M_2 = \sum_{n=1}^{N} M(xM_n)\cos\psi_n \quad (16)$$

for the longitudinal and lateral directions, respectively. Substitution of Eqs. (13) into Eq. (8), together with Eqs. (11, 14, 15, and 16) yield a transfer matrix relationship for longitudinal and lateral cyclic pitch control which follows as Eq. (17).

$$\begin{Bmatrix}\bar{\sigma}_1(s) \\ \bar{\sigma}_2(s)\end{Bmatrix} = \frac{(2K/N)}{(\tau S+1)}\begin{bmatrix}(I_xS^2 + D_x\Omega S + K_x\Omega^2), (2I_x\Omega S + D_x\Omega^2) \\ -(2I_x\Omega S + D_x\Omega^2), (I_xS^2 + D_x\Omega S + K_x\Omega^2)\end{bmatrix}\begin{Bmatrix}\bar{Y}_1(s) \\ \bar{Y}_2(s)\end{Bmatrix} \quad (17)$$

It is seen that longitudinal and lateral cyclic pitch are coupled, interacting controls, and that the control system dynamical behavior is strongly dependent on the initial pitch setting σ_0 through the system parameter K_x.

Discussion

The dynamics of collective pitch changes are governed by Eq. (12) where it is seen that the parameter K_x, the virtual spring constant of the centrifugal force field depends on the inertia difference parameter $(I_z - I_y)$ and the initial pitch setting. Ordinarily z is negligible compared to c, so that this parameter is proportional to $\bar{c}^2\cos 2\sigma_0$. Since σ_0 is of the order of 45° at cruising flight speeds of 250 knots in the propeller state, the character of the dynamics are seen to change from that of a heavily damped oscillator (calculation of typical aerodynamic damping values yield values of the order of 50% of critical) in the hovering helicopter state when σ_0 is a small angle to a cascaded integrator and time constant process.

The dynamics of cyclic pitch change are governed by Eq. (17). Expansion of the system characteristic determinant shows that K_x is the critical parameter. In the hovering helicopter state the transient dynamics are those of a pair of coupled, damped oscillators. As σ_0 approaches 45° in the propeller-rotor state, a divergent oscillation ensues.

It is evident that in the propeller-rotor cruise condition, both collective and cyclic pitch changes should have internal stabilization, the principal component of which would be proportional control action to offset the decreasing trend in K_x with forward speed. In the latter case of cyclic pitch change this is seen to be very important. Other compensation techniques would also be beneficial and would depend on the dynamics of the aircraft itself.

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[2] Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., *Aeroelasticity,* Addison-Wesley, Cambridge, Mass., 1955, pp. 272–279, 282–283.

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Technical Comments

Comment on "Derivation of the Thrust Equation from Conservation of Energy"

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SFORZINI[1] suggests that there are advantages in deriving the thrust equation from energy considerations. The following comments are offered in connection with his paper:

1) Sforzini defines g as the "acceleration resulting from gravity." The energy terms in Eq. (1) and (2) of his paper thus have the dimensions (mass × length) instead of the required dimensions (force × length). Gravitational acceleration is not involved in the thrust equation, however derived.

2) The assumption of inviscid external flow at a uniform pressure p_a leading to a resultant rearward force on the control surface of $F_u + (p_a - p_e)A_e$ is part of the conventional momentum theorem approach, as it is of the energy derivation approach. It has the advantage of focusing attention on the associated definition of drag, i.e. the defined thrust F_u minus the actual forward force delivered by the engine. The drag is thus recognized as the sum of the rearwardly directed force due to the viscosity of the external flow and of the rearwardly directed force due to the gage pressure distribution in the external flow. An important contribution to drag is often made by the gage pressure distribution on the flow boundary upstream of the engine inlet plane, i.e., the additive drag.

3) The inclusion of terms involving f, the fuel-air mixture ratio, has advantages where one wishes to derive the rocket thrust equation from that for an air-breathing engine. In air-breathing engines, however, air bled from the compressor for auxiliary purposes such as turbine-blade cooling closely matches the fuel mass flow rate. The bled air discharges at a low energy level and it is more accurate to account for this loss by neglecting the effect of fuel mass addition than by including it in deriving the thrust equation.

4) Propulsive efficiency is of limited value in propulsion studies. It has a maximum value of unity when the thrust is zero. The over-all efficiency, defined as $(F_u u/\dot{m}_f Q_r)$, where Q_r is the heating value of the fuel, is more useful, since the Breguet range is directly proportional to this quantity. As with all definitions of efficiency it is an energy ratio, however F_u may have been derived.

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Reference

[1] Sforzini, R. H., "Derivation of the Thrust Equation from Conservation of Energy," *Journal of Aircraft,* Vol. 7, No. 6, Nov.-Dec. 1970, pp. 538–540.